



Semester One Examination, 2021

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 1

If required by your examination administrator, please place your student identification label in this box

Section Two: Calculator-assumed

WA student number: In figures

--	--	--	--	--	--	--	--

In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes
 Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
 Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	50	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed

65% (98 Marks)

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9

(9 marks)

The position vector for point A is given by $\underline{a} = 5\underline{i} - 3\underline{j}$, B is given by $\underline{b} = -\underline{i} - 5\underline{j}$ and C is given by $\underline{c} = 8\underline{i} - \underline{j}$. Determine

(a) whether points A, B and C are collinear. (3 mark)

(b) the angle between \underline{a} and \underline{b} (4 marks)

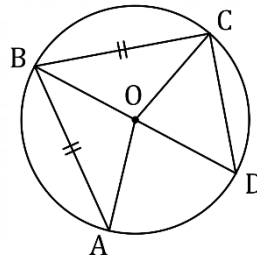
(c) the scalar projection of \underline{b} onto \underline{a} (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 10

(5 marks)

Points A, B, C and D lie on the circle with centre O as shown in the diagram, where $\angle A = 40^\circ$, $AB = BC$ and BD is a diameter.



(a) Determine the size of $\angle AOD$.

(2 marks)

(b) Prove that $\triangle OAD \equiv \triangle ODC$.

(3 marks)

Question 11

(8 marks)

- (a) State whether each of the following statements are true or false, supporting each answer with an example or counterexample.
- (i) A quadrilateral with four congruent sides is a square. (2 marks)
- (ii) The size of one interior angle of a regular polygon with at least five sides is always obtuse. (2 marks)
- (b) Consider the statement $\angle A \geq 90^\circ \Rightarrow \angle B < 90^\circ$ that refers to angles in triangle ABC .
- (i) Write the converse of the statement in simplest form. (1 mark)
- (ii) Write the contrapositive of the statement in simplest form. (1 mark)
- (iii) Briefly discuss the truth of the original statement, the converse statement, and the contrapositive statement. (2 marks)

Question 12

(8 marks)

A small body is acted on by force F_1 of 85 N on a bearing of 260° and by force F_2 of 45 N on a bearing of 025° .

(a) Sketch a diagram to show $F_1 + F_2$ and their resultant R . (2 marks)

(b) Determine the magnitude and bearing of R . (4 marks)

(c) Express R in component form $a\mathbf{i} + b\mathbf{j}$. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 13

(5 marks)

Determine \mathbf{p} , the vector projection of

- (a) a force of 210 N on a bearing 022° onto a force of 300 N on a bearing of 350° . (3 marks)

- (b) \mathbf{v} on \mathbf{w} where $\mathbf{v} = (-16, 63)$ and $\mathbf{w} = (24, -7)$. (2 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 14

(7 marks)

- (a) Points P , Q and R lie on a circle of radius 3 cm, so that PR is a diameter and $PQ = 4$ cm. Determine the exact area of triangle PQR . (3 marks)

- (b) A secant meets a circle at points P and Q , where $PQ = 8$ cm. A tangent to the same circle at point T intersects the secant at point R , where $TR = 12$ cm. Given that $QR < PR$, determine the exact distance PR and the exact distance QR . (4 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 15

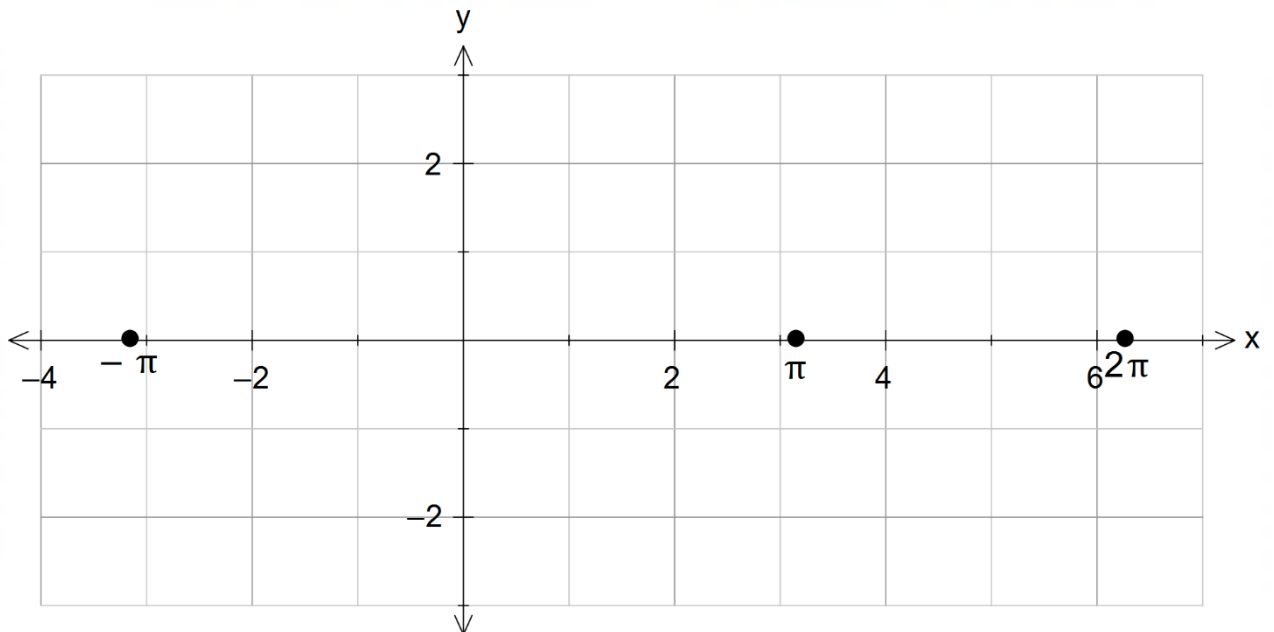
(11 marks)

(a) Solve the following equations.

(i) $\cos(2\theta + \frac{\pi}{6}) = 0.5$ $-\pi \leq \theta \leq \pi$ (4 marks)

(ii) $2\sin 3x = -\sqrt{3}$ (4 marks)

(b) Sketch $y = -2\sin(x + \frac{\pi}{3})$ $-\pi \leq x \leq 2\pi$ on the axes below. (3 marks)



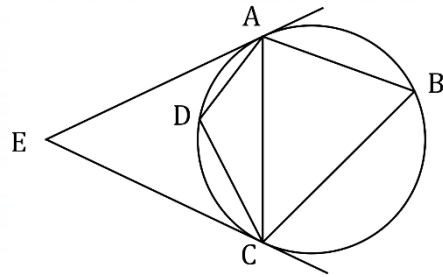
See next page

Question 16

(7 marks)

- (a) The diagram shows points A, B, C and D on the circumference of a circle. Tangents to the circle from A and C meet at point E .

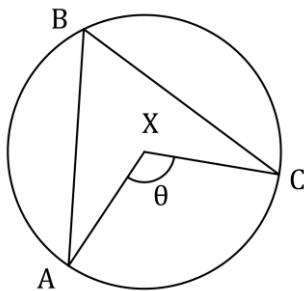
Given that $\angle E = 48^\circ$, determine the size of $\angle B$ and the size of $\angle D$.



(3 marks)

- (b) In the circle shown below $\angle A = 33^\circ$, $\angle C = 28^\circ$ and $\theta = 119^\circ$. Prove by contradiction that X is not the centre of the circle.

(4 marks)



Question 17

(8 marks)

Each letter in the word ACRIMONIOUS is printed individually on a card. When cards are arranged next to each other in a line, determine the number of different permutations

(a) of all the cards. (2 marks)

(b) of all the cards where all the consonants are adjacent. (2 marks)

(c) using any 4 of the cards. (4 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 18

(8 marks)

Small bodies P and Q are moving with constant velocities $(2, -2)$ m/s and $(1, 0)$ m/s respectively.

P has initial position vector $(5, 7)$ m and Q has initial position vector $(-3, 13)$ m.

(a) Determine the distance between the bodies after two seconds. (3 marks)

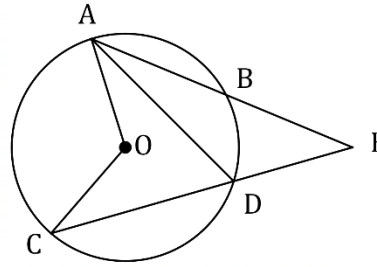
(b) Show that the distance between the bodies after t seconds is given by $\sqrt{5t^2 + 40t + 100}$. (3 marks)

(c) Prove that the bodies do not meet. (2 marks)

Question 19

(7 marks)

In the diagram shown, secants AB and CD intersect at E , a point outside the circle with centre O



- (a) Determine the size of $\angle ADC$ and $\angle AOC$ when $\angle E = 28^\circ$ and $\angle EAD = 22^\circ$. (2 marks)

- (b) Prove that when secants AB and CD intersect at E , a point outside the circle with centre O , then $\angle E = \frac{1}{2}(\angle AOC - \angle BOD)$. (4 marks)

- (c) Determine the size of $\angle E$ when $\angle BOD = 30^\circ$ and $\angle AOC = 80^\circ$. (1 mark)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 20

(7 marks)

- (a) A manufacturer makes the same plastic toy figure in 12 different colours and sells them in packs of three. The toys inside each pack are randomly chosen from the production line in such a way that all are of a different colour.

Determine the least number of packs that a retailer should buy from the manufacturer to be certain of obtaining at least four packs containing the same colour combination of toys.
(3 marks)

- (b) A set of cards is numbered with all the integers from 1 to 15 inclusive. The cards are shuffled, placed face down and then the cards turned over one by one.

Determine how many cards must be turned over to be certain that at least one of the numbers on a face up card will be three times the number on another face up card.
(4 marks)

Question 21

(8 marks)

Harbour Y lies on a bearing of 065° from harbour X and the straight line distance between the harbours is 43 km. Between the harbours, a steady current is moving in a south easterly direction at a speed of 1.5 metres per second.

A boat with a cruising speed of 5.5 metres per second is to travel from harbour X to harbour Y in the least possible time.

- (a) Sketch a diagram, roughly to scale, to show the resultant of the sum of the displacement vectors of the boat and the current. (2 marks)

- (b) Determine the bearing it should steer, to the nearest degree, and the time its journey takes, to the nearest minute. (6 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Supplementary page

Question number: _____

Supplementary page

Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Supplementary page

Question number: _____

Supplementary page

Question number: _____

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

